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Simulation of applied principles of envelope functions for Fabry-Perot spectroscopy of plane wave for single-layer structures

P.S. Kosobutsky

National University "Lviv Polytechnic", 13, Bandery str., 79646 Lviv, Ukraine

E-mail: petkosob@yahoo.com

Abstract. The method to obtain analytical expressions for envelope functions in spectra of normal incidence light reflection and transmission by single-layer structures is proposed.

Keywords: Fabry-Perot interferometry, single-layer structure, reflection, transmission, envelope function method.

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1. Introduction

The principle of multibeam interferometry of light for plane parallel plate developed by Fabry-Perot [1] have been widely used for solving a large number of scientific and applied problems and have been discussed in many textbooks [2-5]. Nevertheless, a number of new regularities in the scope of the problem were recently found. It was shown that Fabry-Perot interferometry is still actual for non-destructive control of optical parameters of micro- and nano-size layers [6-8].

Recently, the so-called method of envelope functions as tangential to contours of amplitude spectra of light reflection and transmission by a single-layer Fabry-Perot structures [10-19] was developed. So, for example, on the basis of the method the approach for determining the variation parameters in thickness and influence of spectral distribution of light and light absorption [9-15] was grounded. The envelope function method was used for grounding the new approach for determination of the instrumental characteristics of interference bands [16, 17] and for reconstruction of the phase of a reflected wave by single-layer films [16, 17, 19]. In other our papers [20-26], we developed the envelope function method for arbitrary ratio between the indices of refraction $n_{1,2,3}$ of three media.

In the paper, authors generalized the basic principles of application of the envelope function method to the data analysis in amplitude-phase spectroscopy of light interference for single-layer structures of Fabry-Perot type (SLSFP) in the case of normal incidence of a beam onto the interfaces for transparent and absorptive structures. A method for

obtaining the analytical expression for energetic coefficients $(R, T)_{\max, \min}$ and phase $\phi_{\max, \min}$ is proposed.

2. General relations

The SLSFP of a thickness d and of the complex refraction index $\tilde{n}_2 = n_2 - i\chi_2$ is bounded by the semi-infinite top transparent media with refraction index n_1 (interface 12) and bottom (transparent or absorbing) media with refraction index \tilde{n}_3 (interface 23). In the absorbing layer, the phase change of wave is equal to $\tilde{\delta} = \frac{4\pi d}{\lambda} \tilde{n}_2$. It is well known also that taking into account the multibeam interference the complex amplitude values of light reflection \tilde{r} and transmission \tilde{t} by SLSFP are determined from

$$\tilde{r} = \frac{\tilde{r}_{12} + \tilde{r}_{23} \exp(-i \tilde{\delta})}{1 + \tilde{r}_{12} \tilde{r}_{23} \exp(-i \tilde{\delta})} \quad \text{and} \quad \tilde{t} = \frac{\tilde{t}_{12} \tilde{t}_{23} \exp(-i \tilde{\delta}/2)}{1 + \tilde{r}_{12} \tilde{r}_{23} \exp(-i \tilde{\delta})}, \quad (1)$$

where $\tilde{r}_{12,23}$ and $\tilde{t}_{12,23}$ are the well-known amplitude Fresnel's coefficients [2] for the single interfaces with subscripts 12 and 23. The resonant dispersion of the media is modelled by one-oscillator function of permittivity [27]

$$\tilde{\epsilon} = \epsilon_0 + \frac{4\pi\lambda \omega_0^2}{\omega_0^2 - \omega^2 + i\omega\tau}. \quad (2)$$

Here ε_0 is the background permittivity, $4\pi\lambda$ is the oscillator force of transition into electron state with resonant frequency ω_0 , τ is the damping factor.

According to Eq. (1), the energy coefficient of reflection R and transmission T , and the tangent of phases $\tan \phi$ and $\tan \Phi$ of light are defined as follows

$$R = \frac{\sigma_{12}^2 + \Theta^2 + 2\sigma_{12}\Theta \cos F_-}{1 + \sigma_{12}^2\Theta^2 + 2\sigma_{12}\Theta \cos F_+},$$

$$T = \frac{n_3 T_{12}T_{23}\Omega}{n_1 (1 + \sigma_{12}^2\Theta^2 + 2\sigma_{12}\Theta \cos F_+)}, \quad (3)$$

and

$$\tan \phi = \frac{\text{Im } \tilde{r}}{\text{Re } \tilde{r}} = \frac{\sigma_{12}(1 - \Theta^2) \sin \phi_{12} + \Theta(1 - \sigma_{12}^2) \sin(\phi_{23} - \text{Re } \tilde{\delta})}{\sigma_{12}(1 + \Theta^2) \cos \phi_{12} + \Theta(1 + \sigma_{12}^2) \cos(\phi_{23} - \text{Re } \tilde{\delta})}, \quad (4)$$

$$\tan \Phi = \frac{\text{Im } \tilde{t}}{\text{Re } \tilde{t}} = \frac{\sin\left(\frac{1}{2}\text{Re } \tilde{\delta}\right) + \sigma_{12}\Theta \sin\left(F_+ - \frac{1}{2}\text{Re } \tilde{\delta}\right)}{\cos\left(\frac{1}{2}\text{Re } \tilde{\delta}\right) + \sigma_{12}\Theta \cos\left(F_+ - \frac{1}{2}\text{Re } \tilde{\delta}\right)} \quad (5)$$

where $\Omega = \exp(-\text{Im } \tilde{\delta})$, $T_{12,23} = \tilde{t}_{12,23} \tilde{t}_{12,23}^*$, $\tilde{t}_{12,23} = \sigma_{12,23} \exp(i\phi_{12,23})$, $F_{\pm} = \phi_{12} \pm (\phi_{23} - \text{Re } \tilde{\delta})$.

Then expression for energetic coefficients of reflection and transmission are defined as [21, 24-26]:

$$R = \frac{R_{\min} + b^2 \cos^2 \frac{F_-}{2}}{1 + b^2 \cos^2 \frac{F_+}{2}} = \frac{R_{\max} - a^2 \sin^2 \frac{F_-}{2}}{1 - a^2 \sin^2 \frac{F_+}{2}} \quad \text{and}$$

$$T = \frac{T_{\min}}{1 - a^2 \sin^2 \frac{F_+}{2}} = \frac{T_{\max}}{1 + b^2 \cos^2 \frac{F_+}{2}}, \quad (6)$$

where $a^2 = \frac{4\sigma_{12}\Theta}{(1 + \sigma_{12}\Theta)^2}$ and $b^2 = \frac{4\sigma_{12}\Theta}{(1 - \sigma_{12}\Theta)^2}$,

$\Theta = \sigma_{23}\Omega$. The functions

$$R_{\max, \min} = \left(\frac{\sigma_{12} \pm \Theta}{1 \pm \sigma_{12}\Theta} \right)^2 \quad \text{and} \quad T_{\max, \min} = \frac{n_3 T_{12}T_{23}\Omega}{n_1 (1 \mp \sigma_{12}\Theta)^2} \quad (7)$$

are the envelope functions of Fabry-Perot spectra.

3. Discussion

1. SLSFP with resonant dispersion (2). In many practical problems, medium with refraction n_1 is air or

vacuum and $n_1=1$. For a free layer with resonant dispersion of $\tilde{\varepsilon}(\omega)$, the calculated spectra $R(\omega)$, $T(\omega)$, $\phi(\omega)$, $\Phi(\omega)$ and their envelope functions $(R, T, \phi)_{\max, \min}$ are shown in Fig. 2. In the region of resonant absorption near the resonant frequency ω_0 , it is possible to separate a frequency interval with width $\Delta\omega_p$, which is bounded by an interval with significant absorption where $\Omega \rightarrow 0$. Inside the interval $R_{\max} \approx R_{\min}$, and the spectra are formed as if the light wave is reflected from a semi-infinite medium with resonant dispersion (2) [21, 24-26]. The distance between envelope functions $\Delta R = R_{\max} - R_{\min}$ and $\Delta T = T_{\max} - T_{\min}$ are equal to

$$\Delta R = \frac{4\sigma_{12}\Theta}{(1 - \sigma_{12}^2\Theta^2)^2} (1 - \sigma_{12}^2)(1 - \Theta^2) \quad \text{and}$$

$$\Delta T = \frac{4\sigma_{12}\Theta\Omega}{(1 - \sigma_{12}^2\Theta^2)^2} \frac{n_3 T_{12}T_{23}}{n_1}. \quad (8)$$

The absorption level increases with approaching to the resonant frequency ω_0 , and the value $\Delta R \rightarrow 0$ because of damping factor approaching to null $\Omega \rightarrow 0$. Near the resonant region beyond the interval $\Delta\omega_p$, the functions (Fig. 1b)

$$\phi_{\max, \min} \cong 2\pi \pm \frac{\sigma_{12}(1 - \sigma_{23}\Theta) \sin \phi_{12} + \sigma_{23}(1 - \sigma_{12}^2)\Omega}{\sigma_{12}(1 + \Theta) \cos \phi_{12}} \quad (9)$$

are the envelope functions of the phase spectrum of reflection. It is problematic to apply the envelope function method (Fig. 1b) to describe the phase spectrum $\Phi(\omega)$ for light transmitted through the layer.

At the arbitrary frequency, we have the following expression:

$$\frac{R_{\max} - R}{R - R_{\min}} = \frac{T - T_{\min}}{T_{\max} - T} = \left(\frac{a}{b} \right)^2 \tan^2 \frac{F_{\pm}}{2}. \quad (10)$$

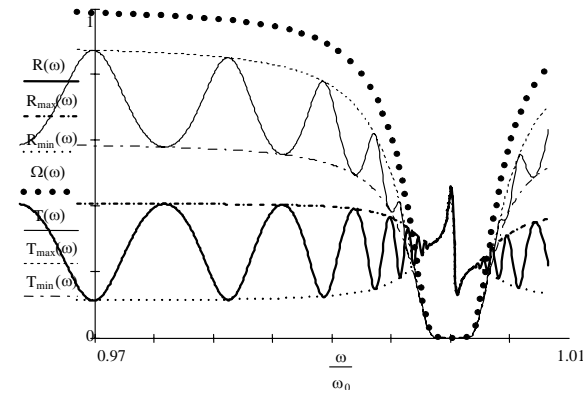
The ratios $\frac{R_{\max} - R}{R - R_{\min}}$ and $\frac{T_{\max} - T}{T - T_{\min}}$ vary from 0 to $+\infty$, and at the frequencies $\omega_{1,2}$ from both side of extreme contour peak the ratios are equal to 1. Hence, coefficients for reflected and transmitted light at the frequencies $\omega_{1,2}$ equal (Fig. 2)

$$\Sigma R = \frac{1}{2} [R_{\max} + R_{\min}] \quad \text{and} \quad \Sigma T = \frac{1}{2} [T_{\max} + T_{\min}]. \quad (11)$$

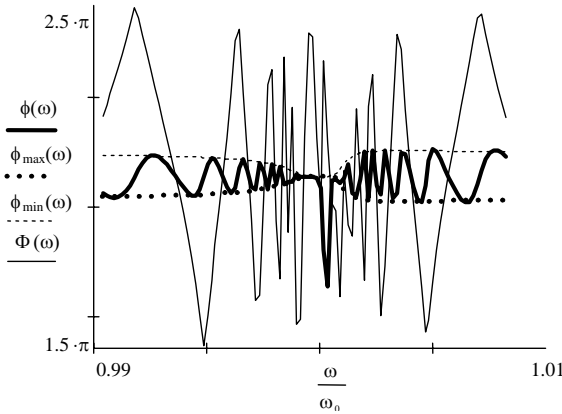
It is the approach (11) that enables to ground the validity of interferogram apparatus characteristic determination [16, 17]. The extreme contour width $\Delta\omega$ under

the condition (11) equals $\Delta\omega = \frac{c_0}{2n_2 d} \Delta\delta$, where $\Delta\delta$ is

the width contour extreme in phase units (phase distance between frequencies $\omega_{1,2}$ on both sides from its peak).



a)



b)

Fig. 1. Calculated spectra of $R(\omega)$, $T(\omega)$ in the resonant region and their envelope functions $(R, T)_{\max, \min}$ (a) and $\phi(\omega)$, $\Phi(\omega)$ (b).

The phase width $\Delta\delta$ equals $\Delta\delta = 2\pi \frac{\Delta\omega}{\Delta\omega_{\min}}$, where $\Delta\omega_{\min}$ is the distance between the frequencies on both sides from the maximum.

The 2π -periodicity of Fabry-Perot spectra allows to determine the area restricted by the contour of a maximum minus the area restricted by envelope function of adjacent minima through the structure parameters as

$$S_R = 2 \int_{\pi/2}^{3\pi/2} [R(\zeta) - R_{\min}] d\zeta, \quad (12)$$

where $\zeta = \frac{\text{Re } \tilde{\delta}}{2}$. For transparent structures the area S_R is equal

$$S_R = (1 - R_{\min}) \left[\pi - \int_{\pi/2}^{3\pi/2} \frac{d\zeta}{1 + b^2 \cos^2 \zeta} \right].$$

This integral is the standard integral [28].

In the region of absorption the integration procedure can be simplified by replacement R_{\min} at the maximum frequency by the mean value of two adjacent

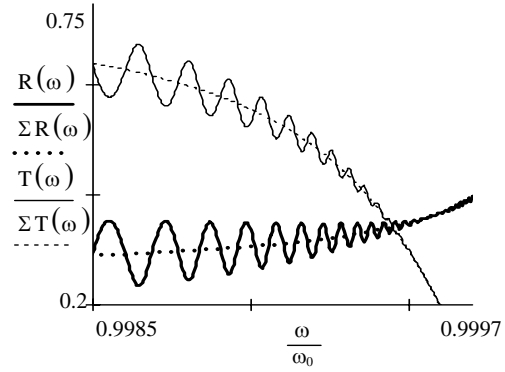


Fig. 2. Calculated spectra of $R(\omega)$, $T(\omega)$ in the resonant region and contours $\Sigma R(\omega)$ and $\Sigma T(\omega)$.

minima $\frac{1}{2}(R_{\min, m-1} + R_{\min, m+1})$ [22]. Clearly, this way can be applied to the transmitted waves.

2. Free layer and the fixed one on substrate surfaces with constant absorption $\chi_2 = \text{const}$.

The values of energetic coefficients $(R, T)_{\max, \min}$ in the absorptive films are a function of λ . Even for absorptive layers with constant absorption, energetic coefficients $(R, T)_{\max, \min}$ vary with variation of λ . According to the definition given by Michelson the visibility of the interferogram $V = \frac{R_{\max} - R_{\min}}{R_{\max} + R_{\min}}$. Then, according to the expression (7) we have

$$V = 2 \left[\frac{\sigma_{12}(1 - \Theta^2)}{(1 - \sigma_{12}^2)\Theta} + \frac{(1 - \sigma_{12}^2)\Theta}{\sigma_{12}(1 - \Theta^2)} \right]^{-1}. \quad (15)$$

Using the transformation

$$2 \frac{x + y}{x - y} = \frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}} + \frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}, \quad (16)$$

we obtained that $V = 2[W^{-1} + W]^{-1}$, where $W = \frac{\sqrt{R_{\max}} - \sqrt{R_{\min}}}{\sqrt{R_{\max}} + \sqrt{R_{\min}}} = \frac{\Theta}{\sigma_{12}} \frac{1 - \sigma_{12}^2}{1 - \Theta^2}$. Therefore, when the

absorption index χ_2 does not depend on frequency, the relative slope between linear dependences $\ln W^{-1}$ and $\text{Im } \tilde{\delta}$ does not depend on frequency for an arbitrary ratio between the refraction indices $n_{1,2,3}$ (Fig. 3) and the relation

$$\ln W^{-1} = \text{const} + \exp(\text{Im } \tilde{\delta}) \quad (17)$$

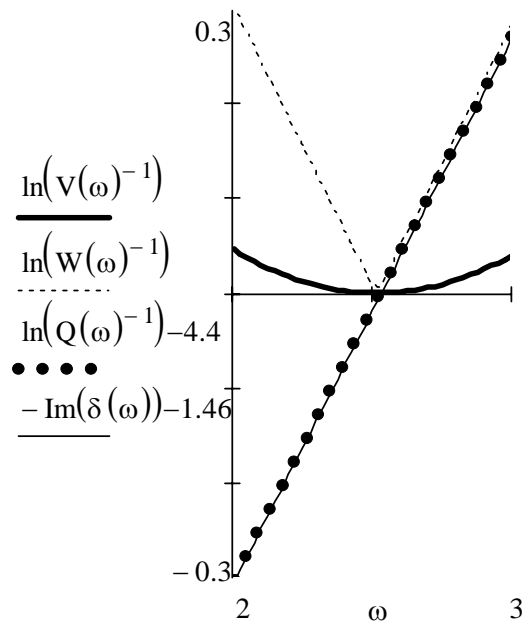


Fig. 3. Calculated dependences of $\ln W^{-1}$, $\ln V^{-1}$ and $\text{Im} \tilde{\delta}$ for parameters: $n_1=1$, $n_2=1.25$, $n_3=3.5$, $\chi_2=0.0038$, $d=15 \mu\text{m}$.

is valid with accuracy to constant. Slope of $\ln V$ depends on the ratio between the refraction indices $n_{1,2,3}$.

4. Principal conclusions

Amplitude-phase Fabry-Perot spectra for single-layer structures at the normal can be described by the envelope functions $(R, T)_{\text{max, min}}$.

The width of the interference band as a phase separation between the points, for which the reflectance are $\frac{1}{2}[(R, T)_{\text{max}} + (R, T)_{\text{min}}]$, is determined through instrumental characteristics of the Fabry-Perot interferogram of single-layer structures.

In the region of constant light absorption in film,

the slope of $\ln \left(\frac{\sqrt{R_{\text{max}}} - \sqrt{R_{\text{min}}}}{\sqrt{R_{\text{max}}} + \sqrt{R_{\text{min}}}} \right)$ equals to that of $\text{Im} \tilde{\delta}$.

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