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Estimation of photodiode frequency characteristics determined by motion of charge carriers in the space charge region. The case of even generation of carriers

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Abstract. Performed in this work is estimation of photodiode frequency characteristics determined by motion of charge carriers in the space charge region for the case of even generation of charge carriers. It has been shown that the current in the external circuit depends on two functions, specific type of which, in its turn, depends on distribution of the electric field and density of generated current, and is fully determined by a set of parameters inherent to photodetector material and incident radiation. The sensitivity for frequencies higher than the boundary one is inversely proportional to the frequency. Simplicity, general form and exactness of the obtained expressions do them suitable for the use when constructing new photodiodes.

Keywords: photodiode, charge carrier, space charge region.

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One of actual tasks in designing pulse optoelectronic devices is exact estimation of characteristics of fast-operating *p-i-n* photodiodes. In particular, in the process of development, on the stages of computations aimed at design of future photodiodes, it is necessary to perform estimation of pulse, frequency characteristics that determine their fast-response. In various works [1-6], some factors determining the kinetics of the diode photoresponse at flight of charge carriers through the space charge region (SCR) were considered along with features of pulse signal detection, construction of pulse photodetectors and their pre-amplifiers. However, the results obtained in these works do not allow to estimate frequency characteristics for developed *p-i-n* photodiodes operating in various modes and conditions for generation of charge carriers.

Several situations when the charge carriers can be generated were considered by us earlier: general case of motion (drift) of charge carriers inside a semiconductor crystal in the electric field of SCR [7], the case of even electric field in the crystal and surface generation of current [8] as well as the case of surface generation of photocurrent at even distribution of the space charge inside the crystal [8].

The analysis of situation of the even electric field and even generation of charge carriers inside the crystal is the purpose of our work, and also situation of even generation of charge carriers and even distribution of the space charge inside the bulk.

The situation of the even electric field and even generation of charge carriers inside the crystal arises when the latter receives comparatively longwave radiation by photodiode with high-ohmic (lightly doped or well compensated) base (clean *p-i-n* photodiode), as a result complete exhaustion of high-ohmic region takes place due to a negligibly low bias voltage as compared to the operation one. In this case, the current transfer is realized by charge carriers of both signs, behaviour of which is almost identical but they move in opposite directions and with different speeds. The total current will be equal to the sum of currents provided by carriers of both signs:

$$I = I_p + I_n . \qquad (1)$$

In the general form, within the limits of assumptions as to the type of modulation of the photosignal that has a sine-like form and described in [7] in detail, the increase of current in the external circuit (photosignal current) is determined according to [7 (1.22)-(1.29)] as

$$d_x\, d_{x'} I = \frac{\vec{E}_x(x)}{\int\limits_0^{x_0} \vec{E}_x(x)\, dx} \times$$
$$\times \sin\omega \left(t - \int\limits_{x'}^{x} \frac{dx''}{\vec{E}_{x''}(x)\bar{\mu}} \right) \cdot \frac{\partial I_0}{\partial x'} \cdot dx' \cdot dx \,, \qquad (2)$$

where dx and dx' are elementary parts of the paths passed by current carriers inside SCR at distances x and x', accordingly; I – photocurrent; E_x – component of the electric field vector along the x-axis; ω – frequency of photosignal modulation; t – considered time moment; μ – mobility of the considered charge carriers; I_0 – effective (internal) amplitude of variable component of generated photocurrent.

It means that the current in the external circuit, in accord with [7], depends on two functions of co-ordinates: electric field and generation density of photosignal current, and the magnitude of this current depends on a specific type of these functions.

Taking into account (2) and the circumstance that we are interested in the situation of even electric field $E_x = E_0 =$ const and even in the whole bulk generation of the photosignal current $\frac{\partial I_0}{\partial x'} = \frac{I_0}{x_0}$, the current of carriers of one sign will be equal:

$$d_x\, d_{x'}\, I = \frac{\vec{E}_x(x)}{\int\limits_0^{x_0} \vec{E}_x(x)\, dx} \cdot \sin\omega\left(t - \int\limits_{x'}^{x} \frac{dx''}{\vec{E}_{x''}(x)\,\vec{\mu}}\right) \times \qquad (3)$$

$$\times \frac{\partial I_0}{\partial x'} \cdot dx' \cdot dx = \frac{I_0}{x_0^2} \cdot \sin\omega\left(t - \frac{x - x'}{\vec{E}_0\,\vec{\mu}}\right) \cdot dx' \cdot dx\,,$$

$$I = \int\limits_0^{x_0} dx' \int\limits_{\vec{x}'}^{0,x_0} \frac{I_0}{x_0^2} \cdot \sin\omega\left(t - \frac{x - x'}{\vec{E}_0\,\vec{\mu}}\right) \cdot dx =$$

$$= \pm I_0 \frac{E_0^2\,\mu^2}{\omega^2 x_0^2}\left[\sin\omega\left(t - \frac{0, x_0 - x_0}{\vec{E}_0\,\vec{\mu}}\right) - \right.$$

$$\left. - \sin\omega\left(t - \frac{0, x_0}{\vec{E}_0\,\vec{\mu}}\right) - \frac{\omega\, x_0}{\vec{E}_0\,\vec{\mu}} \cos\omega t\right] = \qquad (4)$$

$$= \pm I_0 \frac{1}{\omega^2 \tau_n^2}\left[\sin\omega\left(t - \tau_n(0, x_0 - x_0)\right) - \right.$$

$$- \sin\omega\left(t - \tau_n(0, x_0)\right) - \omega\,\tau_n \cos\omega t\,,$$

where: $\bar{\tau}_n = \frac{x_0^2}{V_0\,\mu}$ is the time of flight for carriers passing through the SCR, and the sign "±" takes into account both the direction of carrier movement and limits of integration. For the charge carriers moving in parallel to the axis $\vec{X}$ (from 0 to x_0):

$$I^{\uparrow\uparrow} = I_1\big|_{\vec{v}_x \uparrow\uparrow \vec{x}} = I_0 \frac{1}{\omega^2 \tau_{n1}^2} \times$$
$$\times \left[\sin\omega(t) - \sin(t - \bar{\tau}_{n1}) - \omega\,\bar{\tau}_{n1} \cos\omega t\right]. \qquad (5)$$

For the charge carriers moving in the opposite direction (from x_0 to 0)

$$I^{\uparrow\downarrow} = I_2\big|_{\vec{v}_x \uparrow\downarrow \vec{x}} = -I_0 \frac{1}{\omega^2 \tau_{n2}^2} \times$$
$$\times \left[\sin\omega(t + \bar{\tau}_{n2}) - \sin\omega t - \omega\,\bar{\tau}_{n2} \cos\omega t\right]. \qquad (6)$$

Taking into account that

$$\tau_1 = \frac{x_0}{\vec{E}_0\,\vec{\mu}_1} = \begin{cases} -\tau & \text{at } \vec{v} \uparrow\downarrow \vec{X} \\ \tau & \text{at } \vec{v} \uparrow\uparrow \vec{X} \end{cases}, \qquad (7)$$

$$I_2 = I_0 \frac{1}{\omega^2 \tau_{n2}^2} \cdot \left[\sin\omega t - \sin\omega(t - \tau_{n2}) - \omega\,\tau_{n2} \cos\omega t\right]. \qquad (8)$$

In the general form, outside the dependence on the direction of carrier motion

$$I_{1,2} = I_0 \frac{1}{\omega^2 \tau_{n1,2}^2}\left[\sin\omega t - \sin\omega(t - \tau_{1,2}) - \omega\tau_{1,2} \cos\omega t\right] =$$

$$= I_0 \frac{1}{\omega^2 \tau_{1,2}^2} \cdot \left[\sin\omega t - \sin\omega t \cdot \cos\omega\tau_{1,2} + \right.$$

$$\left. + \cos\omega t \sin\omega\tau_{1,2} - \omega\tau_{1,2} \cos\omega t\right] =$$

$$= I_0 \frac{1}{\omega^2 \tau_{1,2}^2} \cdot \left[\sin\omega t\,(1 - \cos\omega\tau_{1,2}) + \right. \qquad (9)$$

$$\left. + \cos\omega t\,(\sin\omega\tau_{1,2} - \omega\tau_{1,2}\right].$$

With consideration (1)

$$I = I_1 + I_2 = I_0\left[\sin\omega t\left(\frac{1 - \cos\omega\tau_1}{\omega^2 \tau_1^2} + \frac{1 - \cos\omega\tau_2}{\omega^2 \tau_2^2}\right) + \right. \qquad (10)$$

$$\left. + \cos\omega t\left(\frac{\sin\omega\tau_1 - \omega\tau_1}{\omega_2 \tau_1^2} + \frac{\sin\omega\tau_2 - \omega\tau_2}{\omega^2 \tau_2^2}\right)\right].$$

Transforming (10) to the form, we have:

$$I = I_0\,\varphi(\omega, \tau_1, \tau_2) \cdot \sin(\omega t + \Delta\varphi) =$$
$$= I_0 \varphi(\omega, \tau_1, \tau_2) \cdot (\sin\omega t \cos\Delta\varphi + \cos\omega t \sin\Delta\varphi). \qquad (11)$$

We equate coefficients at the identical terms of equations (10) and (11):

$$\begin{cases} \varphi(\omega, \tau_1, \tau_2) \cdot \cos\Delta\varphi = \frac{1 - \cos\omega\,\tau_1}{\omega^2\,\tau_1^2} + \frac{1 - \cos\omega\tau_2}{\omega^2\,\tau_2^2} \\ \varphi(\omega, \tau_1, \tau_2) \cdot \sin\Delta\varphi = \frac{\sin\omega\tau_1 - \omega\tau_1}{\omega^2 \tau_1^2} + \frac{\sin\omega\tau_2 - \omega\tau_2}{\omega^2 \tau_2^2}. \end{cases} \qquad (12)$$

Squaring each term and adding both equations of the system (12), we will get:

$$\varphi^2(\omega, \tau_1, \tau_2) = \left(\frac{1 - \cos\omega\tau_1}{\omega^2 \tau_1^2} + \frac{1 - \cos\omega\tau_2}{\omega^2 \tau_2^2}\right)^2 + \qquad (13)$$

$$+ \left(\frac{\sin\omega\tau_1 - \omega\tau_1}{\omega^2 \tau_1^2} + \frac{\sin\omega\tau_2 - \omega\tau_2}{\omega^2 \tau_2^2}\right)^2.$$

Thereof,

$$I = I_0 \sqrt{\left(\frac{1-\cos\omega\tau_1}{\omega^2\tau_1^2} + \frac{1-\cos\omega\tau_2}{\omega^2\tau_2^2}\right)^2 + \left(\frac{\sin\omega\tau_1 - \omega\tau_1}{\omega^2\tau_1^2} + \frac{\sin\omega\tau_2 - \omega\tau_2}{\omega^2\tau_2^2}\right)^2} \cdot \sin(\omega t + \Delta\varphi). \quad (14)$$

Expressions (10) and (14) describe the dependence of output photosignal current in the external circuit on the frequency of modulation (ω), total time of carrier flight for both signs through SCR $\left(\tau_{1,2} = \frac{x_0^2}{V_0 \mu_{1,2}}\right)$ and the value of generation current (I_0) mainly determined by the power and wavelength of absorbed radiation.

To estimate the value of boundary frequency, we will use GOST 21934-83 (USSR), according to which the boundary frequency is determined from the relation

$$\frac{A(\omega = \omega_b)}{A_{\max}} = \frac{\sqrt{2}}{2}, \quad (15)$$

where $A(\omega = \omega_b)$ is the amplitude of variable component of output current of photosignal in the external circuit at the boundary modulation frequency of radiation; $A_{\max}$ – maximal amplitude of the variable component of output photosignal in the external curcuit.

In our case,

$$A_{\max} = A(\omega = 0) = I_0, \text{ and } \varphi(\omega_b, \tau_1, \tau_2) = \frac{1}{\sqrt{2}}, \quad (16)$$

$$\varphi^2 = \frac{1}{2} = \left(\sqrt{\ldots}\right)^2 = \left(\frac{1-\cos\omega_b\tau_1}{\omega_b^2\tau_1^2} + \frac{1-\cos\omega_b\tau_2}{\omega_b^2\tau_2^2}\right)^2 + \left(\frac{\sin\omega_b\tau_1 - \omega_b\tau_1}{\omega_b^2\tau_1^2} + \frac{\sin\omega_b\tau_2 - \omega_b\tau_2}{\omega_b^2\tau_2^2}\right)^2. \quad (17)$$

Let us designate

$$\omega\tau_1 = Z = \frac{\omega\tau_2}{a} = \omega\tau_n. \quad (18)$$

Then (17) converts to the form:

$$\left(\frac{1-\cos Z}{Z^2} + \frac{1-\cos aZ}{a^2 Z^2}\right)^2 + \left(\frac{\sin Z - Z}{Z^2} + \frac{\sin aZ - aZ}{a^2 Z^2}\right)^2 = \frac{1}{2}, \quad (19)$$

$$\left[a^2(1-\cos Z) + (1-\cos aZ)\right]^2 + \left[a^2(\sin Z - Z) + (\sin aZ - aZ)\right]^2 = \frac{a^4 Z^4}{2}. \quad (20)$$

To solve the equation (20), it is necessary to know the specific a value.

For silicon, for example,

$$\frac{\tau_p}{\tau_n} = a = \frac{\mu_n}{\mu_p} \approx 3. \quad (21)$$

Thereof $Z \approx 1.4$

$$f_b = \frac{\omega_b}{2\pi} = \frac{Z}{2\pi\tau_n} \approx 0.7 \cdot \frac{2V_0 \mu_n}{2\pi x_0^2} = 2.1 \cdot \frac{2V_0 \mu_p}{2\pi x_0^2}. \quad (22)$$

At redesignation $\begin{cases} \tau_1 = \tau_p \\ \tau_2 = \tau_n \end{cases}$ the situation in expression (22) will not change because of identity of terms of motion and generation of charge carriers (and/or because of symmetry of equation (17) in relation to τ_1 and τ_2).

Using (14) and (10), it is always possible, within the limits of the accepted assumptions, to calculate the form of output current signal with the sine-like law of modulation by using the Fourier transformation, and, using (22), it is possible to compare frequency characteristics of different *p-i-n* photodetectors.

Situation of even generation of charge carriers and even density of the space charge inside the crystal arises when receiving the comparatively longwave radiation by the photodiode with a homogeneously doped base at a bias voltage providing the width of the SCR equal to the thickness of the crystal, at the given degree of doping. Like to the previous case when we considered even electric field and even generation of charge carriers inside the crystal, the total current will be equal to the sum of currents for carriers of both signs.

Taking into account the results obtained in [7] and [8 (1.3), (1.5), (2.11), (2.13), (2.14)], the value of the total current can be presented in the form:

$$I = \int_0^{x_0} dx' \int_{x'}^{0,x_0} dx \frac{E_x(x)}{\int_0^{x_0} E_x(x)\,dx} \cdot \sin\omega\left(t - \int_{x'}^{x} \frac{d\vec{x}''}{\vec{E}_x(x)\cdot\bar{\mu}}\right) \frac{\partial I_0}{\partial x'} =$$

$$= \int_0^{x_0} dx' \int_{x'}^{0,x_0} dx \frac{x/\mu\tau}{x_0^3/2\mu\tau} \cdot \sin\omega\left(t - \bar{\tau}\ln\frac{x}{x'}\right)\frac{I_0}{x_0} =$$

$$= \int_0^{x_0} dx' \int_{x'}^{0,x_0} dx \frac{2I_0 x}{x_0^3} \cdot \sin\omega\left(t - \bar{\tau}\ln\frac{x}{x'}\right). \quad (22)$$

With account for replacement of variables in (2.15)-(2.17) of [8]

$$I = \int_0^{x_0} dx' \int_{x'}^{0,x_0} \frac{2I_0}{x_0^2} \cdot \sin\omega u \cdot x' \cdot e^{\frac{t-u}{\bar{\tau}}} \cdot \left(\frac{-1}{\bar{\tau}}\right) x' \cdot e^{\frac{t-u}{\bar{\tau}}} du =$$

$$= \pm\frac{2I}{x_0^3}\left\{-\int_0^{x_0} dx'(0,x_0)^2 \frac{2\sin\omega\left(t - \bar{\tau}\ln\frac{0,x_0}{x'}\right) + \omega\bar{\tau}\cos\omega\left(t - \bar{\tau}\ln\frac{0,x_0}{x'}\right)}{4 + \omega^2\tau^2} + \int_0^{x_0} dx'(x')^2 \frac{2\sin\omega t + \omega\bar{\tau}\cos\omega t}{4 + \omega^2\tau^2}\right\}. \quad (23)$$

We will repeat the replacement with

$$t-\bar{\tau}\ln\frac{0,x_0}{x'}=u, \tag{24}$$

$$x'=(0,x_0)\cdot e^{\frac{u-t}{\bar{\tau}}}, \tag{25}$$

$$dx'=(0,x_0)\cdot e^{\frac{u-t}{\bar{\tau}}}\cdot\frac{du}{\bar{\tau}}, \tag{26}$$

and we will integrate each element of the integral function separately:

$$\int_0^{x_0} dx'\cdot\sin\omega\left(t-\bar{\tau}\ln\frac{0,x_0}{x'}\right)=\int_0^{x_0}\sin\omega u\,(0,x_0)\cdot e^{\frac{u-t}{\bar{\tau}}}\cdot\frac{du}{\bar{\tau}}=$$
$$=x_0\frac{\sin\omega\left(t-\bar{\tau}\ln\frac{0,x_0}{x_0}\right)-\omega\bar{\tau}\cos\omega\left(t-\bar{\tau}\ln\frac{0,x_0}{x_0}\right)}{1+\omega^2\tau^2}, \tag{27}$$

$$\int_0^{x_0} dx'\cdot\cos\omega\left(t-\bar{\tau}\ln\frac{0,x_0}{x'}\right)=\int_0^{x_0}\cos\omega u(0,x_0)\cdot e^{\frac{u-t}{\bar{\tau}}}\cdot\frac{du}{\bar{\tau}}=$$
$$=x_0\frac{\cos\omega\left(t-\bar{\tau}\ln\frac{0,x_0}{x_0}\right)+\omega\bar{\tau}\sin\omega\left(t-\bar{\tau}\ln\frac{0,x_0}{x'}\right)}{1+\omega^2\tau^2}, \tag{28}$$

$$\int_0^{x_0} dx'(x')^2=\frac{x_0^3}{3}. \tag{29}$$

Bearing in mind that

$$(0,x_0)=\begin{cases}0 & \text{at } \vec{v}\uparrow\downarrow\vec{x}\\ x_0 & \text{at } \vec{v}\uparrow\uparrow\vec{x}\end{cases} \tag{30}$$

$$I_1=I_{\uparrow\uparrow}=\frac{2I_0}{x_0^3}\left[\begin{array}{l}\frac{x_0^2}{4+\omega^2\tau_1^2}\cdot\frac{\sin\omega t-\omega\tau_1\cos\omega t}{1+\omega^2\tau_1^2}\cdot 2x+\\ +\frac{x_0^2}{4+\omega^2\tau_1^2}\cdot\omega\tau_1 x_0\frac{\cos\omega t+\omega\tau_1\sin\omega t}{1+\omega^2\tau_1^2}-\\ -\frac{x_0^3}{3}\cdot\frac{2\sin\omega t+\omega\tau_1\cos\omega t}{1+\omega^2\tau_1^2}\end{array}\right]=$$

$$=I_1=I_{\uparrow\uparrow}=\frac{2}{3}I_0\,\frac{\sin\omega t-\omega\tau_1\cos\omega t}{1+\omega^2\tau_1^2}, \tag{31}$$

$$I_2=I_{\uparrow\downarrow}=\frac{2I_0}{x_0^3}\cdot\frac{x_0^3}{3}\cdot\frac{2\sin\omega t+\omega\bar{\tau}_2\cos\omega t}{4+\omega^2\tau_2^2}=$$
$$=\frac{2}{3}I_0\cdot\frac{2\sin\omega t-\omega\tau_2\cos\omega t}{4+\omega^2\tau_2^2}, \tag{32}$$

$$I=I_1+I_2=\frac{2}{3}I_0\left[\frac{\sin\omega t-\omega\tau_1\cos\omega t}{1+\omega^2\tau_1^2}+\right.$$
$$\left.+\frac{2\sin\omega t-\omega\tau_2\cos\omega t}{4+\omega^2\tau_2^2}\right]. \tag{33}$$

Thus

$$\begin{Bmatrix}\bar{\tau}_2=-\tau_2 & \text{when } \vec{v}_2\uparrow\downarrow\vec{x}\\ \bar{\tau}_1=-\tau_1 & \text{when } \vec{v}_1\uparrow\uparrow\vec{x}\end{Bmatrix}. \tag{34}$$

We will reduce (33) to the general form (2.23) in [8] and equate coefficients at the identical terms of equation:

$$\begin{Bmatrix}\varphi(\omega,\tau_1,\tau_2)\cdot\cos\Delta\varphi=\frac{2}{3}\left(\frac{1}{1+\omega^2\tau_1^2}+\frac{2}{4+\omega^2\tau_2^2}\right)\\ \varphi(\omega,\tau_1,\tau_2)\cdot\sin\Delta\varphi=\frac{2}{3}\left(\frac{-\omega\tau_1}{1+\omega^2\tau_1^2}+\frac{-\omega\tau_2}{4+\omega^2\tau_2^2}\right)\end{Bmatrix}. \tag{35}$$

Squaring each term and adding both equations of the system (35), we will get:

$$\varphi^2(\omega,\tau_1,\tau_2)=\frac{4}{9}\left[\left(\frac{1}{1+\omega^2\tau_1^2}+\frac{2}{4+\omega^2\tau_2^2}\right)^2+\right.$$
$$\left.+\left(\frac{\omega\tau_1}{1+\omega^2\tau_1^2}+\frac{\omega\tau_2}{4+\omega^2\tau_2^2}\right)^2\right]= \tag{36}$$
$$=\frac{4}{9}\frac{9+\omega^2(\tau_2+\tau_1)^2}{(1+\omega^2\tau_1^2)(4+\omega^2\tau_2^2)},$$

$$\varphi^2(\omega,\tau_1,\tau_2)=\frac{4}{9}\sqrt{\frac{9+\omega^2(\tau_2+\tau_1)^2}{(1+\omega^2\tau_1^2)(4+\omega^2\tau_2^2)}}, \tag{37}$$

$$I=\frac{2}{3}I_0\sqrt{\frac{9+\omega^2(\tau_2+\tau_1)^2}{(1+\omega^2\tau_1^2)(4+\omega^2\tau_2^2)}}\cdot\sin(\omega t+\Delta\varphi). \tag{38}$$

Exp. (38) describes the dependence of output photosignal current in the external circuit on the frequency of modulation (ω), structural parameters $\left(\tau_i=x_0^2/2V_0\mu_i\right)$ and value of the generated current (I_0) determined, mainly, by the power and wavelength of absorbed radiation.

We will use (2.6) and (2.7) from [8] to estimate the boundary frequency:

$$\frac{A(\omega=\omega_b)}{A(\omega=0)}=\frac{2}{3}\sqrt{\frac{9+\omega_b^2(\tau_2+\tau_1)^2}{(1+\omega_b^2\tau_1^2)(4+\omega_{b0}^2\tau_2^2)}}=\frac{1}{\sqrt{2}}, \tag{39}$$

$$\omega_b=\pm\sqrt{\frac{-(28\tau_1^2+\tau_2^2-16\tau_1\tau_2)\pm\sqrt{(28\tau_1^2+\tau_2^2-16\tau_1\tau_2)^2+4\cdot 36\cdot 9\tau_1^2\tau_2^2}}{2\cdot 9\tau_1^2\tau_2^2}}, \tag{40}$$

but

$$\begin{cases} \text{Re when } \omega \geq 0, \\ \text{Im when } \omega \equiv 0. \end{cases} \quad (41)$$

Therefore,

$$\omega_b = \pm\sqrt{\frac{-(28\tau_1^2+\tau_2^2-16\tau_1\tau_2)\pm\sqrt{(28\tau_1^2+\tau_2^2-16\,\tau_1\tau_2)^2+36^2\tau_1^2\tau_2^2}}{18\tau_1^2\tau_2^2}}. \quad (42)$$

We will indicate: $\tau_1 = a\tau_2$, (43)
then (42)

$$\omega_b = \pm\sqrt{\frac{-(28a^2\tau_2^2+\tau_2^2-16a\tau_2^2)\pm\sqrt{\tau_2^4\,(28a^2+1-16a)^2+36^2a^2\tau_2^4}}{18a^2\tau_2^4}} = \quad (44)$$

$$= \frac{1}{\tau_2}\sqrt{\frac{-(28a^2+1-16a)\pm\sqrt{(28a^2+1-16a)^2+36^2a^2}}{18a^2}}. \quad (45)$$

In the case of silicon, for example,

$$\tau_p \approx 3\tau_n. \quad (46)$$

Exp. (42) is asymmetrical in relation to t_1 and t_2, therefore we will consider two situations corresponding to two opposite polarities of V_0 determined by the type of conductivity of initial material. Let initial silicon has p-type conductivity. Obviously, in this case:

$$\begin{Bmatrix} \tau_1 = \tau_n \\ \tau_2 = \tau_p \\ a \approx \dfrac{1}{3} \end{Bmatrix} \quad (47)$$

and

$$\omega_{b1} = \frac{2.577}{\tau_p} = \frac{0.859}{\tau_n}. \quad (48)$$

When initial silicon has n-type conductivity:

$$\begin{Bmatrix} \tau_1 = \tau_p \\ \tau_2 = \tau_n \\ a \approx 3 \end{Bmatrix} \quad (49)$$

and

$$\omega_{b2} = \frac{0.406}{\tau_n} = \frac{1.218}{\tau_p}. \quad (50)$$

Accordingly, with consideration (2.13) in [8] for silicon of p-type conductivity:

$$f_b^{(p)} = \frac{\omega_{b1}}{2\pi} = 0.859\frac{2V_0\mu_n}{2\pi x_0^2} = 2.577\frac{2V_0\mu_p}{2\pi x_0^2} \quad (51)$$

and for silicon of n-type:

$$f_b^{(n)} = \frac{\omega_{b2}}{2\pi} = 0.406\frac{2V_0\mu_n}{2\pi x_0^2} = 1.218\frac{2V_0\mu_p}{2\pi x_0^2}. \quad (52)$$

Using (38), it is always possible, within the limits of the accepted assumptions, to calculate the form of output current signal with the non-sine-like law of modulation, using the Fourier transformations, and using (51) and (52) it is possible to compare frequency characteristics of different photodetectors.

In all the considered cases, the amplitude of the output signal behaves identically with the change of the modulation frequency in the range of frequencies

$$f << f_b \quad (53)$$

or

$$f >> f_b. \quad (54)$$

Thus,

$$\lim_{f\to 0} I = I_0, \quad (55)$$

$$\lim_{f\to\infty} I = I_0\frac{\text{const}\,(f)}{f}. \quad (56)$$

In principle, that allows to use photodiodes in the frequency band considerably exceeding the value of the boundary frequency, thus for threshold photodetectors operating with high frequencies the threshold of sensitivity grows in proportion to the frequency.

Conclusions

It is shown that the current in the external circuit depends on two functions (their specific form) of co-ordinates of the electric field and density of current generation of photosignal, which are fully determined by the set parameters for photodetector material and received radiation.

Simplicity, general form and exactness of the expressions obtained do them suitable for use in designing the new photodiodes.

Shown is the possibility to use photodiodes at frequencies considerably exceeding their boundary frequency, with respective lowering the current sensitivity and increase in the threshold sensitivity at high frequencies in proportion to frequency.

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